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NUMERICAL MODELING OF AN UNDERWATER EXPLOSION IN AN AIR CAVITY

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An efficient numerical algorithm based on Godunov method is proposed that permits qualitative and quantitative calculations of the hydrodynamic flows resulting from the detonation of explosive charges in an air cavity. Calculations are performed by a difference scheme using moving difference grids in which the moving boundaries are the contact surfaces between the detonation products and air and between air and water, and the shock-wave front. The reliability of the calculations is confirmed by experimental data.

Introduction. Experimental and theoretical studies of the laws of propagation in air and water of shock waves produced by the detonation of condensed high explosives (HE) have been performed by many authors. Experimental results have been presented in numerous publications, foremost in [1–3]. With the development of computer technology and numerical methods, the trend has been toward the theoretical analysis of explosive phenomena [4–6]. As a rule, calculations have been made by methods that permit a through computation with the introduction of artificial viscosity into the difference equations. In this case, the shock front is smeared out over several computation intervals, and the peak values of the hydrodynamic parameters of the front are considerably reduced. Using Lagrangian equations of motion leads to strong deformation of the calculation grid and an increase in calculation time. Using equations in Eulerian variables requires that the problem of determining the position of the contact boundary be solved. The algorithms proposed for this purpose in [7, 8] are very cumbersome and approximate.

Experimental results and an analysis of the character of damping of hydroshock waves in the explosion of HE charges placed at the center of a spherical air cavity are given in [9]. The presence of additional contact boundaries complicates the wave pattern of the process and prevents one from measuring everywhere the parameters being studied. The effective numerical algorithm purposed in the present paper uses Godunov's method of discontinuity decay [10] and allows one to perform a detailed modeling of an underwater explosion in an air cavity. The calculated results are compared with experimental data and confirm the validity and effectiveness of the method used.

1. Formulation of the Problem. We assume that in an unbounded space filled with water at pressure $p = p_1$ and density $\rho = \rho_1$ there is a spherical air cavity of radius $r = R$ at pressure $p = p_1$ and density $\rho = \rho_1^*$. At the center of the latter there is a shell of radius $r = r_0$ containing detonation products (DP) consisting of a highly heated, compressed gas. At the time $t = 0$, the shell disappears. The motions of the DP, air, and water must be calculated.

The nonsteady dynamical behavior of the detonation products and the ambient medium is described by the following system of equations of gas dynamics in Eulerian variables [10]:

$$\frac{\partial \mathbf{a}}{\partial t} + \frac{\partial \mathbf{b}}{\partial r} = \frac{2}{r}(\mathbf{f} - \mathbf{b}); \tag{1.1}$$

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$$\mathbf{a} = \left\| \begin{matrix} \rho \\ \rho u \\ \rho(e + u^2/2) \end{matrix} \right\|, \quad \mathbf{b} = \left\| \begin{matrix} \rho u \\ \rho u^2 + p \\ \rho u(e + u^2/2) + pu \end{matrix} \right\|, \quad \mathbf{f} = \left\| \begin{matrix} 0 \\ p \\ 0 \end{matrix} \right\|. \tag{1.2}$$

Here e is the internal energy per unit mass of gas (liquid) and u is velocity along the r axis.

Equations (1.1) and (1.2) are supplemented by the following equations of state:
(a) for air and the detonation products, $e = p/(æ - 1)\rho$, where the adiabatic exponent for air is $æ = 1.4$ and for the detonation products, it is $æ = æ(\rho)$; formulas for the adiabatic exponent $æ$ of HE of the TNT type are given in [4];
(b) for water, $e = (p + æp_0)/(æ - 1)\rho + c_0^2/(æ - 1)$, where

$$æ = 7.15, \quad p_0 = 3045 \cdot 10^5 \text{ Pa} \quad \text{if} \quad p \leqslant 3 \cdot 10^9 \text{ Pa};$$

$$æ = 6.29, \quad p_0 = 4250 \cdot 10^5 \text{ Pa} \quad \text{if} \quad p > 3 \cdot 10^9 \text{ Pa};$$

$c_0 = \sqrt{(æp_0)/\rho_0}$ is the speed of sound in air under natural conditions.

The model of an actual wave detonation is used. For the spherical case and an explosive of the TNT type, the initial distribution of the thermodynamic parameters of the gas in the region occupied by explosion products, which corresponds to the instant the detonation wave emerges at the surface of the charge, is given by the approximating expressions of [4].

The initial distribution of the parameters of the ambient medium (air and water) is given with allowance for the hydrostatic pressure of the liquid at the known depth of placement of the charge.

2. Method of Numerical Solution. We adopt the traditional notation of Godunov's method of discontinuity decay [10]. For the case of a moving difference grid, the difference scheme approximating the system of differential equations (1.1) and (1.2) can be written as

$$\mathbf{a}^{j-1/2}\Delta r^{j-1/2} = \mathbf{a}_{j-1/2}\Delta r_{j-1/2} - \tau\Big[(\mathbf{B}_j - \mathbf{B}_{j-1}) + \frac{2(\Delta r^{j-1/2} + \Delta r_{j-1/2})}{r^{j-1/2} + r_{j-1/2}}(\mathbf{f} - \mathbf{b})_{j-1/2}\Big]$$

$$(j = 0, 1, \ldots, J),$$

where

$$\mathbf{B}_j = \left\| \begin{matrix} R(U - W) \\ RU(U - W) + P \\ R(U - W)(E + U^2/2) + PU \end{matrix} \right\|_j, \quad \Delta r^{j-1/2} = r^j - r^{j-1},$$

$$\Delta r_{j-1/2} = r_j - r_{j-1}, \quad r^{j-1/2} = 0.5(r^j + r^{j-1}), \quad r_{j-1/2} = 0.5(r_j + r_{j-1});$$

the subscripts correspond to time $t = t_0$ and the superscripts to $t = t_0 + \tau$, P, R, U, and E are values of the hydrodynamic flow parameters p, ρ, u, and e at the nodes of the difference grid calculated by solving the problem of discontinuity decay [10], and W is the velocity of a node.

The allowable step τ of integration with respect to time is given by the formula

$$\tau = \nu \min_j \frac{\Delta r_{j-1/2}}{\max(D_{j-1}^2 - W_j; -D_j^1 + W_{j-1})},$$

in which D_{j-1}^2 and D_j^1 are the velocities of the right-hand and left-hand waves for the discontinuity decay at a node of the difference grid, and ν is the safety margin, $0 < \nu \leqslant 1$.

The universal algorithm of the computer program is constructed under the following assumptions:

(a) the problem is solved in moving grids; the detonation products–air and air–water contact surfaces and the shock front are chosen as the moving boundaries; the intermediate nodes of the difference grid move in accordance with one or another law as a function of the flow regime and the type of medium;

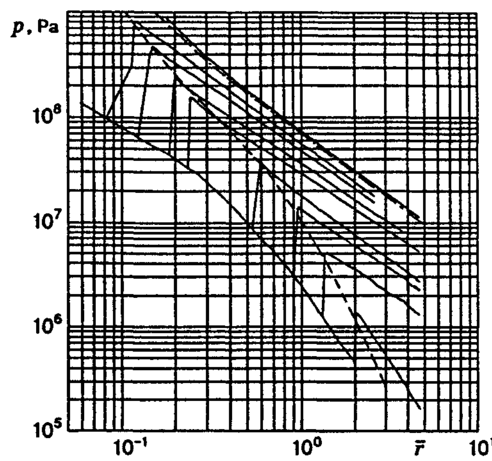

Fig. 1

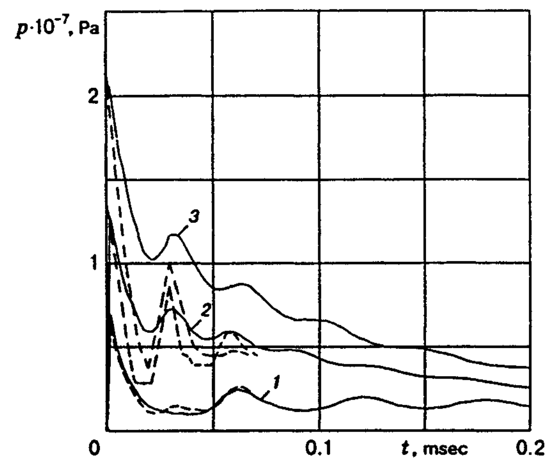

Fig. 2

(b) one calculation cell is taken at the moment the detonation (shock) wave emerges at the contact surface in the ambient medium; in this case, the extreme right-hand node of the grid is identified with the shock front; as the pressure increases in the cell, the shock wave moves away from the contact boundary, leading to an increase in the cell size; the introduction of intermediate grid nodes is provided for in this case (division of the cell whose size becomes larger than a preset amount);

(c) a purely underwater explosion is calculated at $R = r_0$ and the explosion of the HE charge in air is calculated at $R = \infty$.

Thus, everywhere except for narrow zones near the contact surfaces, the calculation is made with isolation of the shock front. This ensures a more reliable calculation of the problem compared to methods of through calculation with the introduction of artificial viscosity.

3. Analysis of the Calculation Results. The mass of the explosive charge was varied in the range $m = (0.42\text{–}2.4) \cdot 10^{-3}$ kg, the radius of the cavity was $R = (1.5\text{–}15) \cdot 10^{-2}$ m, and the ratio of the radii of the cavity and the charge was in the range of $3.75 < R/r_0 < 37.5$.

Following [9], we call an explosion in water a normal explosion. Figure 1 shows the pressure at the shock front as a function of the reduced distance $\bar{r} = r/m^{1/3}$. The dot-and-dashed curve corresponds to the experimental data of [1], the solid upper curve corresponds to a normal explosion, the lower curve to the explosion of a charge in air, and the intermediate curves correspond to explosions in an air cavity for different values of the charge mass m and the cavity radius R.

The start of a sharp pressure rise (intermediate curves) corresponds to the position (reduced coordinate) of the air–water contact surface. In particular, the extreme left-hand curve corresponds to $m = 2.4 \cdot 10^{-3}$ kg and $R = 1.5 \cdot 10^{-2}$ m. The characteristic bend in the rise section is explained by the additional dynamical head produced by the explosion products, which for such m and R follow almost immediately behind the air shock front. The dashed curve denotes the initial pressure at the boundary of the cavity, calculated in [9] on the basis of experimental data; the curve lies near the bend points on the intermediate curves. These points correspond to the pressure at the hydroshock front to the right of the contact surface. Satisfactory agreement between the experimental and calculated results is observed.

Figure 2 shows the time variation of pressure at a distance $r = 0.15$ m from the center of the source for different cavity radii R and charge masses m: $4.37 \cdot 10^{-2}$ m and $0.42 \cdot 10^{-3}$ kg, $1.75 \cdot 10^{-2}$ m and $0.42 \cdot 10^{-3}$ kg, and $1.75 \cdot 10^{-2}$ m and $0.84 \cdot 10^{-3}$ kg (curves 1–3, respectively). The solid curves were plotted from calculated data and the dashed curves are shock-wave oscillograms [9]. Each curve consists of a wave train of decreasing intensity arising in multiple reflection of the air shock wave from the contact surface and the center of symmetry. The amplitudes of the propagating waves and the times of arrival of the reflected waves on the experimental and calculated curves are in good agreement.

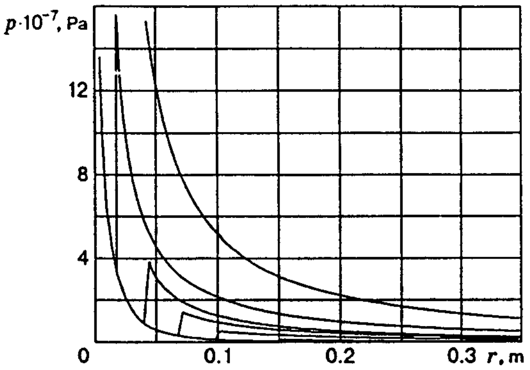

Fig. 3

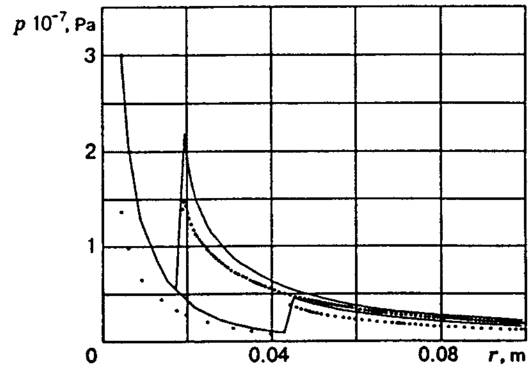

Fig. 4

The pressure at the shock front as a function of distance for $m = 0.42 \cdot 10^{-3}$ kg and different R is shown in Fig. 3. The upper curve corresponds to the normal explosion and the lower curve to the explosion in air. The vertical sections on the intermediate curves correspond to the coordinate of the cavity radius. The maximum pressure at the front of the hydroshock decreases with increase in the cavity radius and becomes considerably lower than the wave pressure in a normal explosion.

The influence of counterpressure in the calculation of explosions in the air cavity on the character of damping of hydroshocks is seen from Fig. 4. The calculations were made for a charge of mass $m = 0.42 \cdot 10^{-3}$ kg for $R = 1.75 \cdot 10^{-2}$ m and $4.37 \cdot 10^{-2}$ m. The dotted curves correspond to $p_1 = 10^5$ Pa and the solid curves to $p_1 = 3 \cdot 10^5$ Pa. It should be noted that a considerable difference between the results obtained for different counterpressures is observed at small distances from the contact surface. This is because the air density increases and the character of formation and damping of the air shock wave changes with increase in pressure in the air cavity. In our case, in particular, the maximum amplitude p of the air shock wave generated by the detonation wave emerging at the contact surface increases from $1.36 \cdot 10^8$ to $3 \cdot 10^8$ Pa.

Experience with numerous calculations made by the authors showed that the safety margin ν takes minimum values at the moments when the shock wave is reflected from the center of the charge and the air–water contact surface, as well as when intermediate nodes are introduced into the difference grid; the value $\nu = 0.25$ always ensures stability of the calculation.

Thus, the numerical algorithm proposed here, based on Godunov's method of discontinuity decay, makes it possible to investigate qualitatively and quantitatively the laws and properties of hydrodynamic flow in explosions of HE charges in an air cavity. The algorithm's reliability is confirmed by the adequacy of the numerical results to the experimental data.

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REFERENCES

1. R. Cole, *Underwater Explosions* [Russian translation], Izd. Inostr. Lit., Moscow (1950).
2. V. D. Khristoforov, "Parameters of the shock wave and gas bubble in underwater explosions of PETN and lead azide charges of different densities," *Prikl. Mekh. Tekh. Fiz.*, No. 4, 118–127 (1961).
3. V. D. Khristoforov, "Parameters of the shock front in air in the explosion of PETN and lead azide charges of different densities," *Prikl. Mekh. Tekh. Fiz.*, No. 6, 175–182 (1961).
4. A. S. Fonarev and S. Yu. Chernyavskii, "Calculation of shock waves in explosions of spherical explosive charges in air," *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 5, 168–174 (1968).
5. L. V. Shurshalov, "Calculation of strong underwater explosions," *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 5, 36–40 (1971).

6. *Underwater and Underground Explosions* (Collected scientific papers) [Russian translation], Mir, Moscow (1974).
7. M. Kh. Abuzyarov and A. V. Kochetkov, "Solution of one-dimensional problems of the hydrodynamics of an explosion in Eulerian variables with isolation of the contact boundaries," in: *Applied Problems of Strength and Plasticity* [in Russian], Izd. Gor'k. Univ., Gor'kii (1985), pp. 66–70.
8. A. I. Kotov, S. K. Matveev, and M. M. Simonenko, "Numerical calculation using Godunov's method of unsteady flows with a contact discontinuity," Leningrad (1985). Deposited at VINITI, 04.11.85, No. 2426-85.
9. V. D. Khristoforov, "An underwater explosion in an air cavity," *Prikl. Mekh. Tekh. Fiz.*, No. 6, 128–132 (1962).
10. S. K. Godunov (ed.), *Numerical Solutions of Multidimensional Problems of Gas Dynamics* [in Russian], Nauka, Moscow (1976).